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14. ABSTRACT
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On the Influence of Internal Density Variations on the Linear Stability Characteristics of Planar Shear Layers

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Planar shear layers are known to be sensitive to density variation within the vorticity region. The current study will expand on the available literature on the stability characteristics of shear layers with density variation through consideration of new regimes in terms of the velocity and density profiles. Both spatial and spatio-temporal linear analyses were considered. In addition to the maximum density change within the shear layer, the effect of density to vorticity thickness ratio, and transverse shifting of the density profile relative to the velocity profile were considered. For a shear layer experiencing combustion, these effects relate to the relative position and thickness of the flame in the shear layer. Thick density profiles have a uniformly stabilizing affect that imposes essentially a low-pass attenuator on the spatial stability. The shear layer is destabilized through the presence of an intense yet thin density reduction that overlaps with the peak vorticity profile, a condition that may be present near the shear layer origination point. The velocity ratio at which the flow transitions between absolute and convective instability is dependent on the magnitude of the density reduction in the shear layer as well as the relative density profile thickness and transverse location. The results generally indicate that the instability of the planar shear layer is strongly influenced by the characteristics of the density-weighted vorticity within the shear layer.

I. INTRODUCTION

Linear stability theory has been found to be a powerful tool for predicting parametric effects of shear flows including the effects of velocity ratio[1-3], density ratio[4-6], heat release[7-9], and compressibility[10-12]. The results of such linear studies provide a theoretical prediction of initial instability frequency or wavelength, the instability growth rate, and the convective/absolute nature of the instability, knowledge that is critical for understanding and controlling shear flows.

A number of linear stability analyses have been applied to reacting shear flows. Shin and Ferziger[7] and Mahalingam, Cantwell, and Ferziger[8] conducted detailed linear stability studies of reacting shear flows including perturbations in heat release. These studies found that the effects of heat release were accurately replicated when only accounting for the base velocity and density profiles. In further support of this approximation, Day, Reynolds, and Mansour[11] suggest that the mean density-weighted vorticity profile is a strong indicator of instability for compressible reacting shear flows. It is apparent that this approximation is likely to breakdown for reacting flows near quenching boundaries[13].

The primary objective of the current research was to investigate the affects of heat release in the shear layer region on the convective/absolute instability transition. Experimental studies indicate that crossing from convective into absolute instability is associated with the emergence of a self-excited state that experiences rapid spatial growth, entrainment, and energetically-enhanced turbulence[14-16]. Although heat release in general has a stabilizing effect on the shear layer, and reduces entrainment[17-22], there is an opportunity to use linear theory to identify reacting shear layer configurations that are absolutely unstable. A reacting shear layer in an absolute instability regime will likely have enhanced entrainment characteristics, providing an opportunity to increase combustion intensity. Additionally, absolutely unstable flows, if properly tuned, have potential use in combustion control applications to avoid thermo-acoustic instabilities. This analysis will also promote understanding of the role of flame position and flame thickness relative to the shear layer on the instability characteristics. In particular, the flame thickness relative to the shear layer may be very different in the flame stabilization region compared to the fully-developed turbulent regime located further downstream.

We denote the two-dimensional base flow velocity components, density and pressure by $\overline{U}$, $\overline{V}$, $\overline{\rho}$ and $\overline{p}$, and the corresponding quantities for the disturbance by u', v', ρ' and p'. The problem can be simplified by the assumption of parallel base flow, i.e. $\overline{U} = \overline{U}(y)$, $\overline{V} = 0$, $\overline{\rho} = \overline{\rho}(y)$ and $\overline{p} = const$. The results of Day et al.[11] indicate that at low convective Mach numbers, the shear layer with heat release will be dominated by two-dimensional instabilities, thus three-dimensional instability was not considered.

The solution to the linear disturbance equations can be decomposed into normal modes[23]. The transverse perturbation velocity term, for example, becomes of the form

$$v'(x,y,t) = \hat{v}(y)\ \exp\left[i(\alpha x - \omega t)\right], \tag{1}$$

where α is the wavenumber and ω is the frequency. The complex wave speed is defined as $c = c_r + i c_i = \dfrac{\omega}{\alpha}$.

The substitution of the normal modes into the governing equations and reducing the system of equations to a single equation in terms $\hat{v}$ yields

$$\overline{\rho}\left[(\overline{U}-c)\left(\frac{d^2\hat{v}}{dy^2}-\alpha^2\hat{v}\right)-\frac{d^2\overline{U}}{dy^2}\hat{v}\right]+\frac{d\overline{\rho}}{dy}\left[(\overline{U}-c)\frac{d\ \hat{v}}{dy}-\frac{d\overline{U}}{dy}\hat{v}\right]=0. \tag{2}$$

For the planar shear layer, as $y \to \pm\infty$, equation (2) simplifies to the form

$$\left(\frac{d^2\hat{v}}{dy^2}-\alpha^2\hat{v}\right)=0. \tag{3}$$

which goes to zero exponentially as $y \to \pm\infty$.

Equation (2) together with the boundary conditions (Eq. 3) and base profiles form the stability model for two-dimensional incompressible parallel shear layer with density variation. The system of equations is solved using the shooting method.

II. Shear Layers with Internal Density Variation

The shear layer velocity ratio is defined as $\lambda = \dfrac{U_1 - U_2}{U_1 + U_2}$, where U_1 and U_2 are the high and low-speed freestream velocities, respectively. The velocity profile is in the form of a hyperbolic tangent shape, the most common model for analyzing free shear layers[24]

$$U(y) = (\Delta U/2)\tanh\left(2y/\delta_\omega\right) + \overline{U}\ , \tag{4}$$

where ΔU is the velocity difference across the layer, $\overline{U}$ is the average velocity, and δ_ω is the vorticity thickness defined as

$$\delta_\omega = \frac{\Delta U}{U'_{\max}}, \tag{5}$$

where $U'_{\max}$ is the maximum slope of the mean velocity.

In order to study the stability of shear layers with heat release, computations are developed for shear layers with a density reduction within the shear layer region. In addition to considering the intensity of the density change, the relative thickness and transverse location of the density reduction were also considered. The density profile used for this purpose is

$$\rho(y) = 1 + A \tanh\left(B\, y - 0.1\right) - A \tanh\left(B\, y + 0.1\right) \tag{6}$$

where $A = \dfrac{1-s}{2\tanh(0.1)}$ and s is the ratio of the minimum/maximum density to the free stream density,

$$s = \frac{\rho_0}{\rho_\infty}. \tag{7}$$

The thickness of the density profile is adjusted by varying the parameter *B*. A measure used to assess the thickness of the density profile is the density thickness, δ_ρ. This quantity is defined as

$$\delta_\rho = 2\frac{\Delta\rho}{\rho'_{\max}}, \tag{8}$$

where $\Delta\rho$ is the magnitude of the density change. The variable r is used to represent the relative density thickness and is defined as

$$r = \frac{\delta_\rho}{\delta_\omega}. \tag{9}$$

The assumed profile shapes are used to facilitate a general parametric study of the influence of heat release on a planar shear layer. Real planar flames will exhibit deviations from the assumed shape, including the presence of asymmetry in the velocity and/or density profiles. In general, the results of linear stability with assumed profile shapes have been predictive of the results of

real experimental flows, as demonstrated by Strykowski and coworkers[14-16, 25]. It is thus anticipated that the results will capture the general trends of the affects of heat release.

III. Influence of the Density Ratio s

The spatial analysis has been performed for the base velocity and density profiles shown in Figs. 1 and 2, respectively. In Fig. 2, density profiles are plotted versus the vorticity magnitude profile

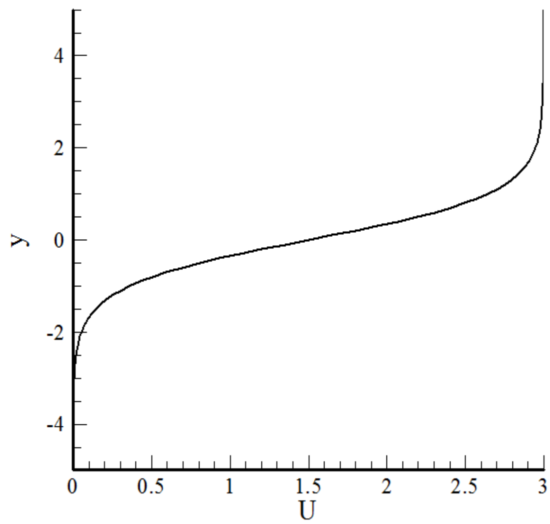

Figure 1. Base velocity profile $U(y) = 1.5\tanh(y) + 1.5$

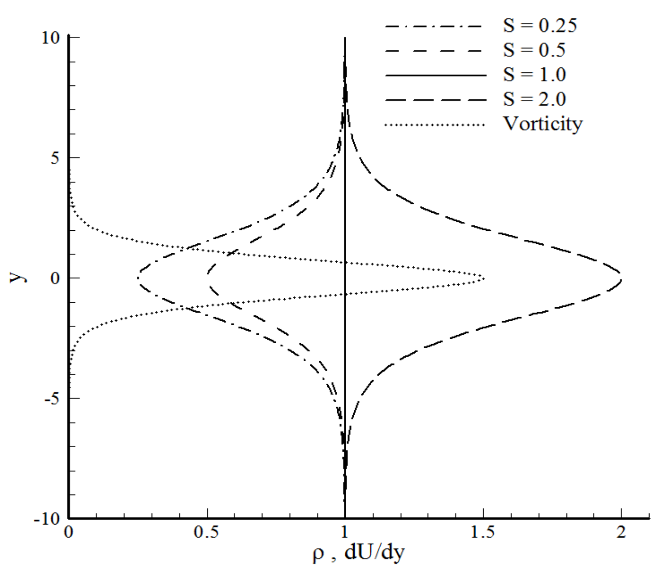

Figure 2. Base density profile versus the vorticity magnitude profile for internal density variation for $r = 3.0$

for a relative density thickness $r = 3.0$. The dimensionless spatial growth rate, $-\alpha_i^*$, versus the dimensionless frequency ω^*, is illustrated in Fig. 3 for different density ratios. The dimensionless frequency and growth rate are defined as

$$\omega^* = \frac{\omega \delta_\omega}{\Delta U} \tag{10}$$

and

$$\alpha_i^* = \alpha_i \delta_\omega . \tag{11}$$

An increase in density, which is destabilizing, is also shown for reference. The figure indicates

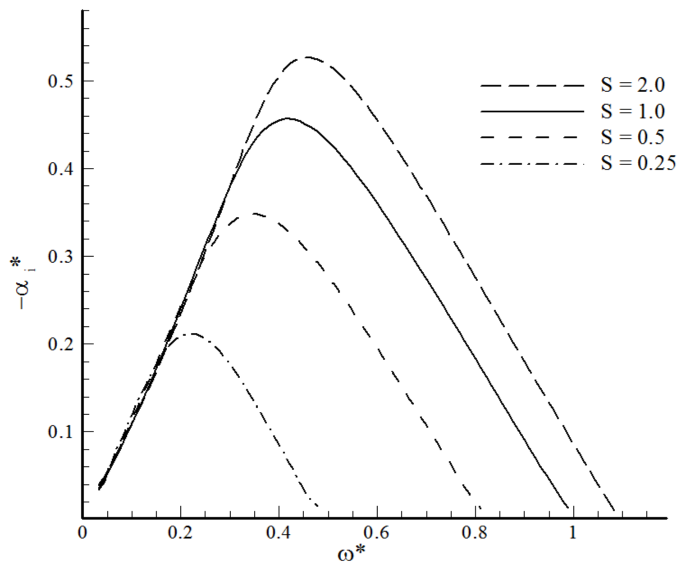

Figure 3. Spatial growth rate for internal density variation for $r = 3.0$

the stabilizing effect of reducing the density ratio. The low-frequency side of the growth rate profile is nominally unchanged by the presence of heat release until the peak is approached. It is also apparent that the high-frequency side of the growth rate profile experiences a constant reduction in amplification rate as indicated by the parallel curves. The stabilizing effect of heat release is essentially a low-pass attenuator with a cutoff frequency that depends on the density ratio, and an asymptotically constant reduction magnitude at high frequency; the damping effect does not grow with increase in frequency at high frequency. For density ratios above unity, the density variation acts as a low-pass amplifier of instability.

The same analysis was done for a case with the same velocity profile but a density profile corresponding to a thickness ratio r = 1.0. This base flow may be expected to be associated with the region near the splitter-plate trailing edge of a reacting shear layer under conditions where the combined boundary layer thicknesses of the two streams is larger than the diffusion flame thickness. Although r is unity, Fig. 4 shows that the density variation is indeed thinner than the

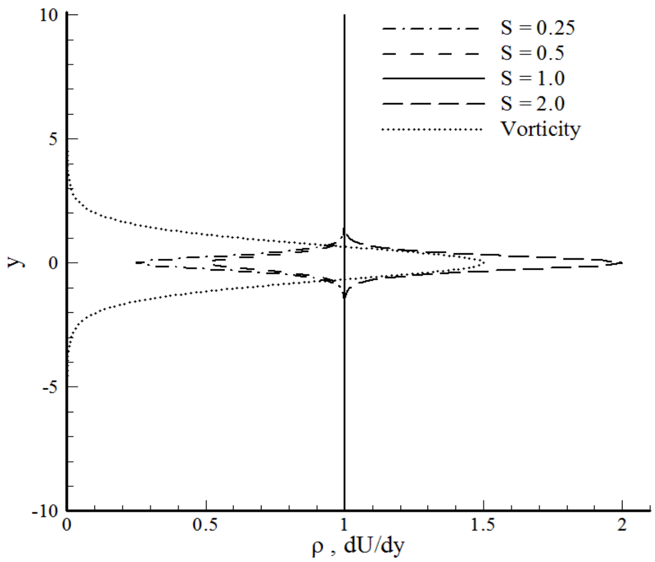

Figure 4. Base density profile versus the vorticity magnitude profile for internal density variation for $r = 1.0$

vorticity variation. Figure 5 clearly demonstrates a similar stabilizing trend with decreasing density ratio except for s = 0.25. By decreasing the density ratio from 0.5 to 0.25, the peak spatial growth rate increases dramatically. The case for s = 0.5 shows some increase in instability at low frequencies that was not seen in Fig. 3. It is clear from Fig. 5 that the low-pass attenuator affect changes for the thinner density profile for sufficiently low density ratio. This transition correlates to changes in the shape of the density-weighted vorticity profile, which will be described in the next section.

IV. Influence of the Thickness Ratio

The nonmonotonic behavior illustrated in Fig. 5 clearly relates to the relative thickness of the density profile, since it is not observed in the results for the thicker density profile (Fig. 3). Therefore, the next step was to investigate the affect of the relative thickness on the stability of the flow. Figure 6 shows the amplification rate of the most unstable mode at s = 0.5 and 0.25 for

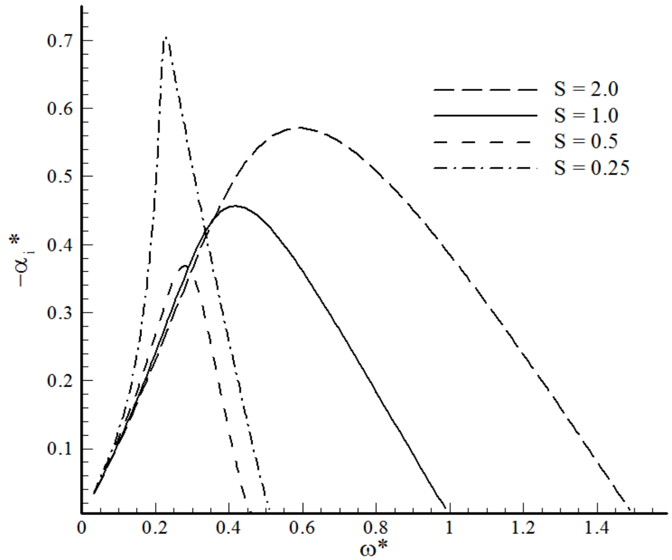

Figure 5. Spatial growth rate for internal density variation for $r = 1.0$

different values of the thickness ratio r. It is very clear from the figure that the influence of heat release on the stability of the shear layer is complex. At sufficiently high values of r, the heat release is clearly stabilizing (growth rates are smaller than the uniform density case). There does exist a range of r at low values where the heat release is actually destabilizing. Further insight into this is gained through consideration of the density-weighted vorticity profile.

As suggested by Day et al., the density-weighted vorticity plays a key role in the instability characteristics of shear layers.[11] At a fixed (but less than unity) density ratio, the density-weighted vorticity profile may have two different shapes. At large r, the density weighted-vorticity profile exhibits a single centered peak. In this regime, the affects of heat release are uniformly stabilizing. At small r, the density-weighted vorticity profile will exhibit two peaks located symmetrically away from the center of the shear layer. These two regimes are indicated on Fig. 6. In the two-peak regime, the heat release can have a destabilizing effect. This regime is also associated with low-frequency amplification as shown in Fig. 5; the low-pass attenuator behavior is associated with the single peak regime.

Figure 7 shows the peak magnitude of the density-weighted vorticity as a function of r and s. In the single peak regime, the peak amplitude of the density-weighted vorticity does not depend on the thickness ratio. In the two-peak regime, a reduction in thickness ratio actually results in

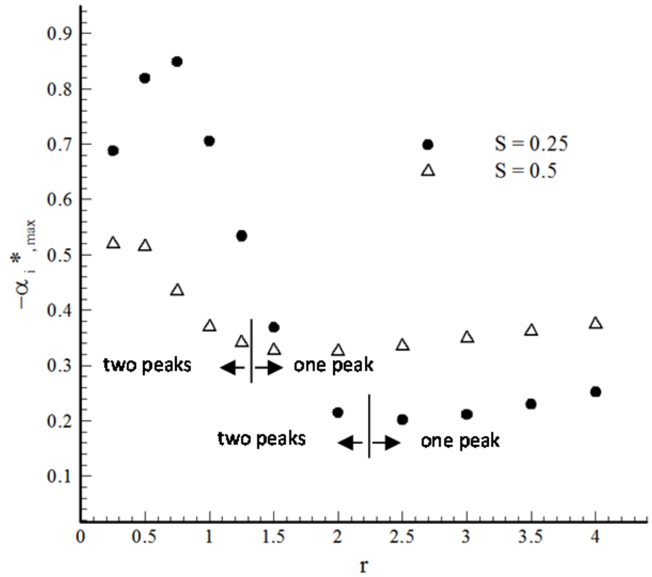

Figure 6. Spatial amplification rate of the most unstable mode as a function of *r*

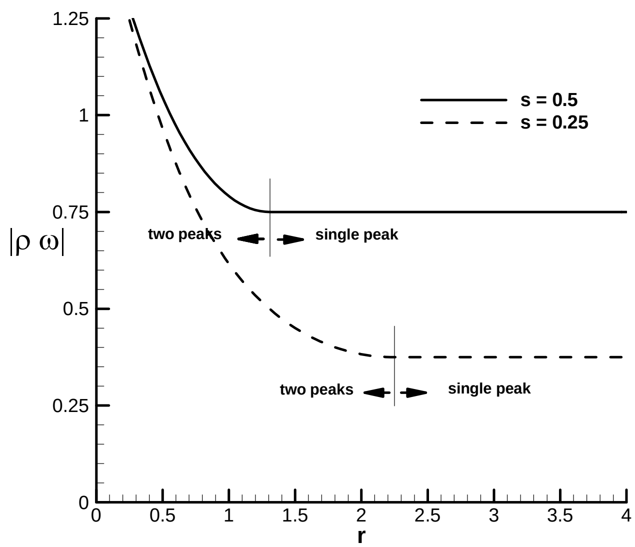

Figure 7. Peak density-weighted vorticity as a function of *r*

higher amplitudes of the peak density-weighted vorticity, which according to Day et al. suggests enhanced instability. This is consistent with the trend shown in Figs. 5 and 6. The results of Figs. 6 and 7 further suggest the utility of the density-weighted vorticity profile as an indicator of instability. Although the peak density-weighted vorticity for s = 0.25 is always lower than that

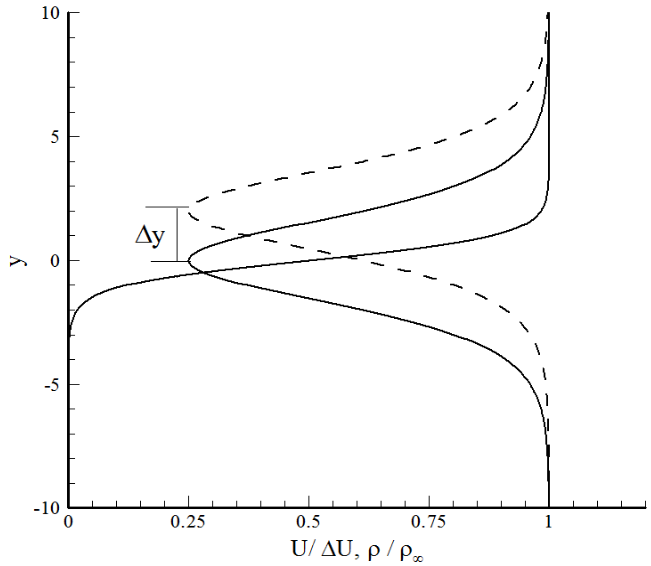

Figure 8. Density profile shift with respect to the velocity profile for $r = 2.0$

for s = 0.5, very thin density layers for s = 0.25 are much more unstable than for s = 0.5, as shown in Fig. 6. For these very thin layers, the smaller density ratio s will tend to push the somewhat lower amplitude peak further away from the center of the shear layer. Thus the overall instability is dependent both on the peak magnitude in the density-weighted vorticity as well as the transverse location of the peak(s). Further transverse displacement of the vorticity peak appears to result in enhanced instability.

V. Influence of Shifting the Density Profile

In addition to the influence of density layer thickness, the affect of transverse shifting of the density profile was explored. The analysis consists of shifting the density profile while keeping the velocity profile fixed. For a shear layer experiencing combustion, the shift relates to the position of the flame in the shear layer and is illustrated by Fig. 8 for r = 2.0. Physically, the shifting of the heat release location in the shear layer is related to the lateral location of the stoichiometric mixture fraction, which is governed by the reactant concentrations in the freestreams and the entrainment ratio. Shifts towards both the high- and low-speed streams were considered. The magnitude of the displacement is expressed in relation to the vorticity thickness, δ_{ω}. Figure 9 shows the amplification rate for different offsets of the density profile for r = 3.0

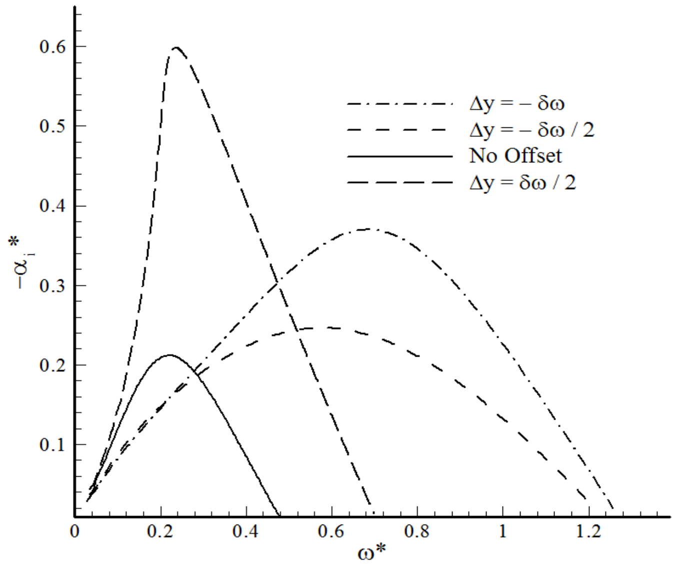

Figure 9. Spatial growth rate for internal density variation with shifted density profile, $r = 3.0$, $s = 0.25$

and s = 0.25. This case is in the single density-weighted vorticity peak regime. As the density profile is moved towards the upper or lower stream, the flow becomes more unstable. It should be noted here that shifting the density profile in this single peak regime of the density-weighted vorticity results in a higher peak value (slightly skewed away from the shifting direction), thus suggesting enhanced instability. It is apparent that shifting the density-weighted vorticity towards the low-speed side has a more dramatic destabilizing influence. Results for shifting other thickness ratio s cases may be found in Hajesfandiari[26].

VI. Spatio-temporal Instability

The idea of absolute and convective instability was first introduced by Briggs[27] and Bers[28] in the study of plasma instabilities. The absolute-convective nature of the instability is identified when there exists at least one point, where $\partial\omega/\partial\alpha = 0$, in the complex α plane. This point is a singularity point in the wavenumber space and a saddle point in the frequency space.

In one of the initial applications of this concept towards fluid mechanics, Huerre and Monkewitz[29] applied the spatio-temporal analysis to hyperbolic tangent shear layers with uniform density. They solved the stability equation for different values of velocity ratio to explore the affect of this parameter on the absolute-convective transition of the instabilities.

Based on their findings, the zero group velocity mode with zero ω_i , i.e. $\omega_i^0 = 0$, occurs when $\lambda = 1.315$. This critical velocity ratio corresponds to $U_2/U_1 = -0.136$. This implies that absolute instability may start to emerge in the shear layer in the presence of a sufficiently high counterflow. Strykowski and Niccum[15] experimentally studied the counterflowing mixing layer for different velocity ratios. The critical value of the velocity ratio obtained by them was $\lambda = 1.32$ which agrees strongly with linear theory.

In the present work, exploring the role of the internal density variation on the absolute-convective instability transition of free shear layers is of interest. For this purpose, the spatio-temporal analysis is applied to different flow profiles in order to find out at what value of velocity ratio the transition from convective to absolute instability occurs in the presence of internal density variations. Additionally, the effect of the density thickness and transverse offset on the transition velocity ratio was also studied.

Figure 10 shows the critical velocity ratio at which the instability characteristics switch from

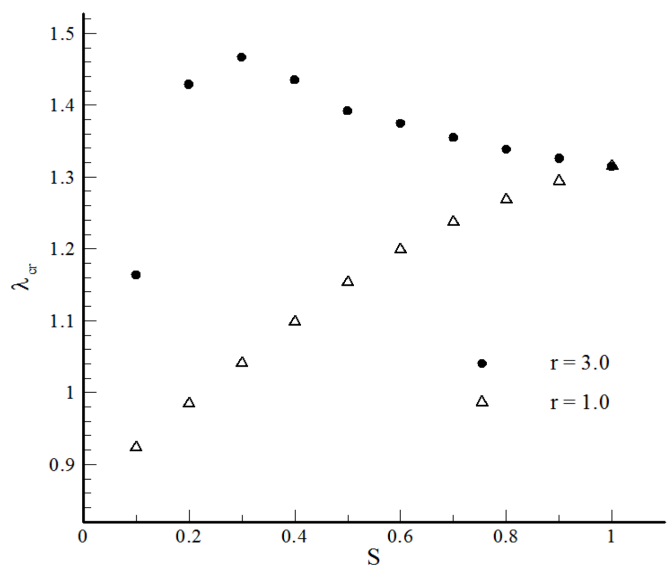

Figure 10. Critical velocity ratio as a function of density ratio

absolute to convective. The results are represented for two density profiles that are different in thickness. For thick density layers, absolute instability is inhibited (i.e. requires higher levels of counterflow) except at very low density ratios, while the trend for the thinner density layer

appears to promote absolute instability with decreasing density ratio. The trend of a thinner layer promoting absolute instability is in agreement with the findings of Reynal et al.[30] for low-density planar jets. For density ratios below 0.25, the critical velocity ratio for r = 1 is smaller than unity, suggesting absolute instability without the requirement of counterflow. This scenario has potential applicability to the flame anchoring region, either attached to the trailing edge or lifted, where the flame and/or flammable region may be thinner than the vorticity layer of the velocity profile. The vorticity and density thickness in the downstream region are more likely to be the same due to turbulent transport (visually this would be associated with r ~ 2), thus the flame may have limited impact on the transition velocity ratio. In general, the results suggest that local regions of absolute instability may be promoted or abated depending on the relative flame thickness, but the global flame which may include large regions of thickness ratio near two will have approximately the same tendency to be absolutely unstable (thus globally unstable) as a nonreacting shear layer.

The effect of transverse shifting of the density profile on the absolute-convective transition of the instability was also explored. The value of the critical velocity ratio with shifting of the density profile to the high and low-speed streams is illustrated in Fig. 11 for r = 3.0. When the density

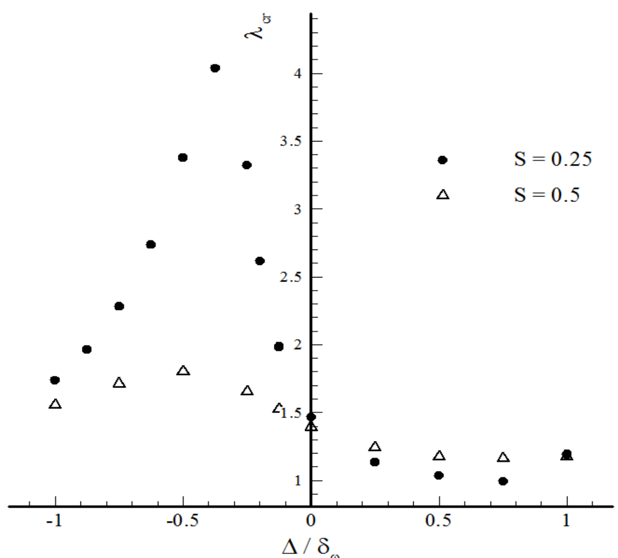

Figure 11. Critical velocity ratio as a function of transverse offset when $r = 3.0$

profile is shifted to the low-speed side, absolute instability is less prevalent in that higher levels of counterflow are required. This stabilizing effect of shifting the density profile to the low-

speed side is consistent with the results of Reynal et al.[30] and Srinivasan, Hallberg, and Strykowski[31] for low density jets. Divergent from the trend shown by Reynal et al., shifting the density profile to the high-speed side results in enhancement of absolute instability. Similar effects are observed for different thickness ratios and can be found in Hajesfandiari .[26]

X. SUMMARY

In general, the results indicate that the influence of a layer of heat release on a planar shear layer can have a wide range of affects on the hydrodynamic instability. When the density layer is relatively thick compared to the vorticity thickness, a reduction in density in the shear layer has a stabilizing effect in terms of spatial growth. If the layer is shifted towards one of the freestreams, it appears that the stabilizing effect is somewhat degraded, and the flow can be highly unstable when the heat release is shifted to the high-speed side. The affect of heat release is essentially a low-pass attenuator that degrades amplification above a cutoff frequency that depends on density ratio.

For the case of a relatively thin density profile, the reduction of the density to a sufficient level results in an eventual destabilization of the flow, with growth rates that can exceed the uniform density case. This destabilization effect degrades if the layer is shifted away from the center of the shear layer.

The above-stated conclusions further support the utility of the density-weighted vorticity as an indicator of instability. The peak value within the density-weighted vorticity profile is sensitive to the thickness ratio when two peaks are present in the profile, and the instability appears to also be very sensitive to thickness ratio r in this regime. In the range of profile characteristics where a single peak is present, the peak density-weighted vorticity is constant with changing thickness ratio, and the instability is very weakly dependent on the thickness ratio. In this regime, the thickness ratio causes very slight changes in the profile shape of the density-weighted vorticity. One should be cautioned to not place too much emphasis on the density-weighted vorticity. The instability characteristics will be very much dependent on the freestream conditions as well as the behavior of the density-weighted vorticity profile. One case in point, the instability of a hyperbolic tangent velocity profile with a freestream density difference will be highly dependent on whether the low-density side is on the high or low velocity side of the shear layer. These cases would have nominally the same density-weighted vorticity profile. Nonetheless, processes

internal to the shear layer that alter the distribution of the density-weighted vorticity can have dramatic consequences on the stability without changes in the freestream conditions.

The results provide insight on the instability of different spatial regions of reacting shear layers. In general, the turbulent region of the shear layer will be associated with a density profile that is of similar thickness to the velocity profile, due to the governing nature of the turbulent transport. Exceptions would include the case where the stoichiometric mixture fraction is dramatically different from the entrainment ratio of the shear layer, which would lead to a flame that resides on the outside (or inside) edge of the shear layer, or exhibiting asymmetry.[17-19] If the flame is located nominally within the shear layer and has similar thickness, the flame tends to stabilize the shear layer, although transition to absolute instability occurs at a similar level of counterflow. This provides encouragement that global instability may be achievable in reacting shear layers with realizable counterflow levels. It is apparent that different behaviors, including higher growth rates and lower transitional velocity ratios, are present when the density profile is thinner than the velocity-based shear layer profile. This affect for thin flames is very sensitive to the transverse location of the flame. The results in general support the need for further exploration of employing global instabilities in reacting flows to enhance mixing and combustion processes.

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